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Nondestructive Testing and Target Identification

David Colton
UNIVERSITY OF DELAWARE

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14. ABSTRACT
This grant is concerned with three problems that are related to Air Force needs: 1) The extension of the linear sampling method in target identification from the frequency domain to the time domain, 2) the development of the theory of transmission eigenvalues in nondestructive testing to include the cases of materials involving thin layers and multiple small inhomogeneities, and 3) the use of transmission eigenvalues in nondestructive testing with applications to airplane canopies and related problems. The main accomplishments during the period of this report were:
1. The investigation of the linear sampling method in the time domain for limited aperture data and for objects in a waveguide.
2. The use of asymptotic methods to investigate the transmission eigenvalue problem for inhomogeneous media that contains small penetrable homogeneous inclusions and thin layers and the use of linear sampling methods to detect the delamination of materials.
3. The use of Stekloff eigenvalues in the nondestructive testing of anisotropic, absorbing inhomogneous media.

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Final Performance Report

September 1, 2013 - October 31, 2016

Principal Investigators

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Objectives

This grant is concerned with three problems that are related to Air Force needs: 1) The extension of the linear sampling method in target identification from the frequency domain to the time domain, 2) the development of the theory of transmission eigenvalues in nondestructive testing to include the cases of materials involving thin layers and multiple small inhomogeneities, and 3) the use of transmission eigenvalues in nondestructive testing with applications to airplane canopies and related problems.

Accomplishments/New Findings

The main accomplishments during the period of this report were:

1. The investigation of the linear sampling method in the time domain for limited aperture data and for objects in a waveguide.

2. The use of asymptotic methods to investigate the transmission eigenvalue problem for inhomogeneous media that contains small penetrable homogeneous inclusions and thin layers and the use of linear sampling methods to detect the delamination of materials.

3. The use of Stekloff eigenvalues in the nondestructive testing of anisotropic, absorbing inhomogneous media.

Status of Efforts

The linear sampling method has become a popular method for solving a variety of problems in target identification for both acoustic and electromagnetic waves. Until recently this method was restricted to problems in the frequency domain. However, a drawback of the frequency domain linear sampling method is the large amount of data required for its implementation and the failure of the method for small aperture data sets. In an effort to

overcome the shortcomings of the frequency domain linear sampling method, recent progress has been made on developing a linear sampling method in the time domain. In [15] we gave an example where the linear sampling method for a limited number of sources and receivers at a fixed frequency fails but succeeds in the time domain. Further examples in [15] show that the limitation of the frequency domain linear sampling method can be alleviated to a considerable extent by going to the time domain. This investigation was continued in [24] where time domain linear sampling methods were used to identify a penetrable object situated in a waveguide.

Transmission eigenvalues have come to play an increasingly important role in inverse scattering theory and a survey of recent results in this area can be found in the CBMS-NSF sponsored monograph [18]. In a variety of problems in nondestructive testing, transmission eigenvalues can be used to obtain information about the location, refractive index or the size of small inhomogeneities from the measured real transmission eigenvalues for the perturbed media and the computed transmission eigenvalues for the unperturbed media. The idea was developed in [9] with particular attention being given to the case when the unperturbed media is inhomogeneous. The analysis in [9] was based on the development of asymptotic expansions for transmission eigenvalues. In [7] an asymptotic analysis of the transmission eigenvalue problem was performed for conducting media coated by an thin layer of a dielectric. In a different but related direction, the linear sampling method was coupled together with asymptotic analysis in [20] to detect whether two materials that should be in contact have separated or delaminated. Such problems occur frequently in composite structures, concrete and other engineering applications and the results of [20] present a new approach to this problem.

The use of transmission eigenvalues in nondestructive testing as described above has two major drawbacks. The first drawback is that in general only the first transmission eigenvalue can be accurately determined from the measured data and the determination of this eigenvalue means that the frequency of the investigating wave must be varied in a frequency range around this eigenvalue. In particular, multifrequency data must be used in an a priori determined frequency range. This also requires the medium to be non-dispersive. The second drawback is that only real transmission eigenvalues can be conveniently determined from the measured scattering data which means that transmission eigenvalues cannot be used for the nondestructive testing of inhomogeneous absorbing media. These difficulties were overcome in [22] through the use of Stekloff eigenvalues instead of transmission eigenvalues although the theory in this case is not as complete as it is for transmission eigenvalues.

The above studies of delamination, and the use of Stekloff eigenvalues in nondestructive testing were for the case of acoustic waves. The extension of these ideas to the case of electromagnetic waves is ongoing.

Personnel Supported

1. Faculty:
   D. Colton, F. Cakoni and P. Monk (Principal Investigators)

2. Short - Term Visitors:
   July 2014 Marc Bonnet
   Sept 2014: Haddar Houssem
   Nov 2014: Alexandra Smimova
   Dec 2014: Drossos Gintides
   May 2015: Rainer Kress
   Nov 2015: Valery Serov
   March 2016: Virginia Selgas
   October 2016: Shuonan Wu

3. Graduate Students:
   2015 Summer:
   De Teresa Trueba, Irene
   Harris, Isaac
   Kapita, Shelvean
   Yang, Fan

   2016 Summer:
   Cogar, Sam
   Civiletti, Ben
   De Teresa Trueba, Irene
   Kapita, Shelvean

Interactions/Transitions

Professors Colton, Monk and Cakoni have attended numerous conferences and seminars as invited speakers both in this country and Europe and Asia.

Archival publications (published) during reporting period

2014

1. F. Cakoni, D. Colton and S. Meng, The inverse scattering problem for a penetrable cavity with internal measurements, *AMS Contemporary Mathematics*, **615**, 71-88 (2014).

   Abstract: We consider the inverse scattering problem for a cavity that is bounded by a penetrable inhomogeneous medium of compact support and seek to determine the shape of the cavity from internal measurements on a curve or surface inside the cavity. We prove uniqueness and establish a linear sampling method for determining the shape of the cavity. A central role in our analysis is played by an unusual non-selfadjoint eigenvalue problem which we call the exterior transmission eigenvalue problem.

2. F. Cakoni, Y. Hu and R. Kress, Simultaneous reconstruction of shape and generalized impedance functions in electrostatic imaging, *Inverse Problems*, **30**, paper 105010 (2014)

Abstract: Determining the geometry and the physical nature of an inclusion within a conducting medium from voltage and current measurements on the accessible boundary of the medium can be modeled as an inverse boundary value problem for the Laplace equation subject to appropriate boundary conditions on the inclusion. We continue the investigations on the particular inverse problem with a generalized impedance condition started in Cakoni and Kress (2013 Inverse Problems 29 015005) by presenting an inverse algorithm for the simultaneous reconstruction of both the shape of the inclusion and the two impedance functions via a boundary integral equation approach. In addition to describing the reconstruction algorithm and illustrating its feasibility by numerical examples we also provide some extensions to the uniqueness results in Cakoni and Kress (2013 Inverse Problems 29 015005).

3. F. Cakoni, P. Monk and J. Sun, Error analysis of the finite element approximation of transmission eigenvalues, *Computational Methods in Applied Mathematics*, **14**, 419-427, (2014).

   Abstract: In this paper we consider the transmission eigenvalue problem corresponding to acoustic scattering by a bounded isotropic inhomogeneous object in two dimensions. This is a non-self-adjoint eigenvalue problem for a quadratic pencil of operators. In particular we are concerned with theoretical error analysis of a finite element method for computing the eigenvalues and corresponding eigenfunctions. Our analysis of convergence makes use of Osborns perturbation theory for eigenvalues of non-self-adjoint compact operators. Some numerical examples are presented to confirm our theoretical error analysis.

4. I. Harris, F. Cakoni and J. Sun, Transmission eigenvalues and non-destructive testing of anisotropic magnetic materials with voids, *Inverse Problems*, **30**, paper 035016 (2014).

   Abstract: In this paper we consider the transmission eigenvalue problem corresponding to the scattering problem for an anisotropic magnetic materials with voids, i.e. subregions with refractive index the same as the background. Here we restrict ourselves to the scalar case of TE-polarization. Under weak assumptions on the material properties, we show that the transmission eigenvalues can be determined from the far field measurements. Then assuming that the contrast on the material properties does not change sign, we prove the existence of at least one transmission eigenvalue for sufficiently small voids. We also show that the first transmission eigenvalue can be used to determining material properties and give qualitative information about the size of the void. Some numerical examples are given to demonstrate the theoretical results.

5. S. Meng, H. Haddar and F. Cakoni, The factorization method for a cavity in an inhomogeneous medium, *Inverse Problems*, **30**, paper 045008 (2014).

   Abstract: We consider the inverse scattering problem for a cavity that is bounded by a penetrable anisotropic inhomogeneous medium of compact support and seek to determine the shape of the cavity from internal measurements on a curve or surface inside

the cavity. We derive a factorization method which provides a rigorous characterization of the support of the cavity in terms of the range of an operator which is computable from the measured data. The support of the cavity is determined without a priori knowledge of the constitutive parameters of the surrounding anisotropic medium provided they satisfy appropriate physical as well as mathematical assumptions imposed by our analysis. Numerical examples are given showing the viability of our method.

6. Y. Hu, F. Cakoni and J. Liu, The inverse problem for a partially coated cavity with interior measurements, *Applicable Analysis*, **93**, 936-956 (2014).

   Abstract: We consider the interior inverse scattering problem of recovering the shape and the surface impedance of an impenetrable partially coated cavity from a knowledge of measured scatter waves due to point sources located on a closed curve inside the cavity. First, we prove uniqueness of the inverse problem, namely, we show that both the shape of the cavity and the impedance function on the coated part are uniquely determined from exact data. Then, based on the linear sampling method, we propose an inversion scheme for determining both the shape and the boundary impedance. Finally, we present some numerical examples showing the validity of our method

2015

7. F. Cakoni, N. Chaulet and H. Haddar, Asymptotic analysis of the transmission eigenvalue problem for a Dirichlet obstacle coated by a thin layer of non-absorbing media, *IMA J. Appl. Math*, **80**, 1063-1098, (2015).

   Abstract: We consider the transmission eigenvalue problem for an impenetrable obstacle with Dirichlet boundary condition surrounded by a thin layer of non-absorbing inhomogeneous material. We derive a rigorous asymptotic expansion for the first transmission eigenvalue with respect to the thickness of the thin layer. Our convergence analysis is based on a MaxMin principle and an iterative approach which involves estimates on the corresponding eigenfunctions. We provide explicit expressions for the terms in the asymptotic expansion up to order 3.

8. F. Cakoni and I. Harris, The factorization method for a defective region in an anisotropic material, *Inverse Problems*, **31**, paper 025002 (2015).

   Abstract: In this paper we consider the inverse acoustic scattering (in $\mathbb{R}^3$) or electromagnetic scattering (in $\mathbb{R}^2$, for the scalar TE-polarization case) problem of reconstructing possibly multiple defective penetrable regions in a known anisotropic material of compact support. We develop the factorization method for a non-absorbing anisotropic background media containing penetrable defects. In particular, under appropriate assumptions on the anisotropic material properties of the media we develop a rigorous characterization for the support of the defective regions from the given far field measurements. Finally we present some numerical examples in the two-dimensional case

to demonstrate the feasibility of our reconstruction method including examples for the case when the defects are voids (i.e. subregions with refractive index the same as the background outside the inhomogeneous hosting media).

9. F. Cakoni, S. Moskow and S. Rome, Perturbations of transmission eigenvalues for inhomogeneous media in the presence of small inclusions, *Inverse Problems and Imaging*, **9**, 725 - 748, (2015).

   Abstract: This paper concerns the transmission eigenvalue problem for an inhomogeneous media of compact support containing small penetrable homogeneous inclusions. Assuming that the inhomogeneous background media is known and smooth, we investigate how these small volume inclusions affect the real transmission eigenvalues. Note that for practical applications the real transmission eigenvalues are important since they can be measured from the scattering data. In particular, in addition to proving the convergence rate for the eigenvalues corresponding to the perturbed media as inclusions' volume goes to zero, we also provide the explicit first correction term in the asymptotic expansion for simple eigenvalues. The correction terms involves the eigenvalues and eigenvectors of the unperturbed known background as well as information about the location, size and refractive index of small inhomogeneities. Thus, our asymptotic formula has the potential to be used to recover information about small inclusions from a knowledge of real transmission eigenvalues.

10. F. Cakoni, H. Haddar and H. Harris, Homogenization of the transmission eigenvalue problem for periodic media and application to the inverse problem, *Inverse Problems and Imaging*, **9**, 1025 - 1049, (2015).

    Abstract: We consider the interior transmission problem associated with the scattering by an inhomogeneous (possibly anisotropic) highly oscillating periodic media. We show that, under appropriate assumptions, the solution of the interior transmission problem converges to the solution of a homogenized problem as the period goes to zero. Furthermore, we prove that the associated real transmission eigenvalues converge to transmission eigenvalues of the homogenized problem. Finally we show how to use the first transmission eigenvalue of the period media, which is measurable from the scattering data, to obtain information about constant effective material properties of the periodic media. The convergence results presented here are not optimal. Such results with rate of convergence involve the analysis of the boundary correction and will be subject of a forthcoming paper.

11. F. Cakoni, H. Haddar and S. Meng, Boundary integral equations for the transmission eigenvalue problem for Maxwell's equations, *J. Int. Eqns. Appl.*, **27**, 377-406, (2015).

    Abstract: In this paper, we consider the transmission eigenvalue problem for Maxwell's equations corresponding to non-magnetic inhomogeneities with contrast in electric permittivity that changes sign inside its support. We formulate the transmission eigenvalue problem as an equivalent homogeneous system of the boundary integral equation

and, assuming that the contrast is constant near the boundary of the support of the inhomogeneity, we prove that the operator associated with this system is Fredholm of index zero and depends analytically on the wave number. Then we show the existence of wave numbers that are not transmission eigenvalues which by an application of the analytic Fredholm theory implies that the set of transmission eigenvalues is discrete with positive infinity as the only accumulation point.

12. D. Colton, Y.-J. Leung and S. Meng, Distribution of complex transmission eigenvalues for spherically stratified media, *Inverse Problems*, **31**, paper 035006 (2015).

    Abstract: In this paper, we employ transformation operators and Levinson's density formula to study the distribution of interior transmission eigenvalues for a spherically stratified media. In particular, we show that under smoothness condition on the index of refraction that there exist an infinite number of complex eigenvalues and there exist situations when there are no real eigenvalues. We also consider the case when absorption is present and show that under appropriate conditions there exist an infinite number of eigenvalues near the real axis.

13. D. Colton. Y.-J. Leung and S.X. Meng, The inverse spectral problem for exterior transmission eigenvalues, *Inverse Problems*, **30**, paper 055010 (2014).

    Abstract: We consider the exterior transmission eigenvalue problem for spherically stratified media in R-3 and consider the case of axially symmetric eigenfunctions. The exterior transmission eigenvalue problem is then reduced to a problem in ordinary differential equations. We first determine conditions on the index of refraction which guarantee the existence of infinitely many complex eigenvalues or infinitely many real eigenvalues. We then show that if two sets of spectral data are known, then under appropriate conditions the index of refraction is uniquely determined.

14. D. Colton and S.X. Meng, Spectral properties of the exterior transmission eigenvalue problem, *Inverse Problems*, **30**, paper 105010 (2014).

    Abstract: The exterior transmission eigenvalue problem arises naturally when one considers the scattering of point sources situated in a cavity by the penetrable non-absorbable boundary of the cavity. Here we show that for constant index of refraction the exterior transmission eigenvalues form a discrete set and for the case of a spherically stratified medium the eigenvalues (both real and complex) uniquely determine the index of refraction.

15. Y. Guo, P. Monk and D. Colton, The linear sampling method for sparse small aperture data, *Applicable Analysis*, **95**, 1599-1615 (2015).

    Abstract: In an effort to improve the performance of the linear sampling method in situations involving sparse data-sets, this method in inverse scattering has recently been extended from the frequency domain to the time domain. In this paper, we consider

the relative merits of the time and multifrequency linear sampling methods for sparse, limited aperture, and data-sets. Among our conclusions are that, for limited aperture measurements single-frequency data can fail to reconstruct the scatterer, whereas both time and multifrequency domain data perform satisfactorily. On the other hand, if the aperture is too small all the sampling methods fail and increasing the number of measurements in a fixed size aperture is of no help.

16. A. Lechleiter and P. Monk, The time-domain Lippmann-Schwinger equation and convolution quadrature, *Numerical Methods for Partial Differential Equations*, **31**, 517-540 (2015).

    Abstract: We consider time-domain acoustic scattering from a penetrable medium with a variable sound speed. This problem can be reduced to solve a time-domain volume Lippmann-Schwinger integral equation. Using convolution quadrature in time and trigonometric collocation in space, we can compute an approximate solution. We prove that the time-domain Lippmann-Schwinger equation has a unique solution and prove conditional convergence and error estimates for the fully discrete solution for globally smooth sound speeds. Preliminary numerical results show that the method behaves well even for discontinuous sound speeds.

17. J. Li, P. Monk and D. Weile, Time domain integral equation methods in computational electromagnetism, in *Computational Electromagnetism*, Lecture Notes in Mathematics, 2148 (CIME Foundation Subseries). A. Bermúdez de Castro and A. Valli Eds, 111-189 (2015).

    Abstract: A tutorial chapter on convolution quadrature based numerical methods for discretizing time domain integral equations arising in computational electromagnetism.

2016

18. F. Cakoni, D. Colton and H. Haddar, *Inverse Scattering Theory and Transmission Eigenvalues*, CBMS-NSF Regional Conference Series in Applied Mathematics, **88**, SIAM Publications, 2016.

    Abstract. This monograph is a survey of recent developments in inverse scattering theory based on the linear sampling method, the factorization method and the theory of transmission eigenvalues. The authors begin with a basic introduction to inverse scattering theory and then proceed to a detailed discussion of the transmission eigenvalue problem. In addition to the well-known linear sampling and factorization methods, the authors present a new generalized linear sampling method which highlights the relationship between these two methods. For the sake of clarity of presentation, focus is made on the scalar inverse scattering problem for inhomogeneous media.

19. F. Cakoni and R. Kress, A boundary integral equation method for the transmission eigenvalue problem, *Applicable Analysis*, **96**, 23-38 (2016).

    Abstract: We propose a new integral equation formulation to characterize and compute transmission eigenvalues for constant refractive index that play an important role in inverse scattering problems for penetrable media. As opposed to the recently developed approach by Cossonnire and Haddar which relies on a two by two system of boundary integral equations our analysis is based on only one integral equation in terms of Dirichlet-to-Neumann or Robin-to-Dirichlet operators which results in a noticeable reduction of computational costs. We establish Fredholm properties of the integral operators and their analytic dependence on the wave number. Further we employ the numerical algorithm for analytic non-linear eigenvalue problems that was recently proposed by Beyn for the numerical computation of transmission eigenvalues via this new integral equation.

20. F. Cakoni, I. De Teresa, H. Haddar and P. Monk, Nondestructive testing of the delaminated interface between two materials, *SIAM J. Appl. Math.*, **76**, 2306-2332, (2016).

    Abstract: We consider the problem of detecting whether two materials that should be in contact have separated or delaminated. The goal is to find an acoustic technique to detect the delamination. We model the delamination as a thin opening between two materials of different acoustic properties, and using asymptotic techniques we derive an asymptotic model where the delaminated region is replaced by jump conditions on the acoustic field and flux. The asymptotic model has potential singularities due to the edges of the delaminated region, and we show that the forward problem is well posed for a large class of possible delaminations. We then design a special linear sampling method (LSM) for detecting the shape of the delamination assuming that the background or undamaged state is known. Finally, we show by numerical experiments that our LSM can indeed determine the shape of delaminated regions.

21. F. Cakoni D. Colton and J. Rezac, The Born transmission eigenvalue problem, *Inverse Problems* **32** paper 105014 (2016).

    Abstract: In this paper we study the distribution of transmission eigenvalues in the complex plane for obstacles whose contrast is small in magnitude. We use a first order approximation of the refractive index to derive and study an approximate interior transmission problem. In the case of spherically stratified media, we prove existence and discreteness of transmission eigenvalues and derive a condition under which the complex part of transmission eigenvalues cannot lie in a strip parallel to the real axis. For obstacles with general shape, we demonstrate that if transmission eigenvalues exist then they form a discrete set.

22. F. Cakoni, D. Colton, S. Meng and P. Monk, Steklov eigenvalues in inverse scattering, *SIAM J. Appl. Math.* **76**, 1737-1763 (2016).

Abstract: We consider a problem in nondestructive testing in which small changes in the (possibly complex valued) refractive index $n(x)$ of an inhomogeneous medium of compact support are to be determined from changes in measured far field data due to incident plane waves. The problem is studied by considering a modified far field operator $\mathcal{F}$ whose kernel is the difference of the measured far field pattern due to the scattering object and the far field pattern of an auxiliary scattering problem with the Stekloff boundary condition imposed on the boundary of a domain B, where B is either the support of the scattering object or a ball containing the scattering object in its interior. It is shown that $\mathcal{F}$ can be used to determine the Stekloff eigenvalues corresponding to B, where, if $B \neq D$, the refractive index is set equal to one in $B \setminus \overline{D}$. A formula is obtained relating changes in $n(x)$ to changes in the Stekloff eigenvalues and numerical examples are given showing the effectiveness of determining changes to the refractive index in this way.

23. F. Cakoni, B. Guzina and S. Moskow, On the homogenization of a transmission problem in scattering theory for highly oscillating media, *SIAM J. Math. Analysis*, **48**, 2532-2560, (2016).

    Abstract: We study the homogenization of a transmission problem arising in the scattering theory for bounded inhomogeneities with periodic coefficients modeled by the anisotropic Helmholtz equation. The coefficients are assumed to be periodic functions of the fast variable, specified over the unit cell with characteristic size ϵ. By way of multiple scales expansion, we focus on the $O(\epsilon^k)$, $k = 1, 2$, bulk and boundary corrections of the leading-order ($O(1)$) homogenized transmission problem. The analysis in particular provides the H^1 and L^2 estimates of the error committed by the first-order-corrected solution considering (i) bulk correction only and (ii) boundary and bulk correction. We treat explicitly the $O(\epsilon)$ boundary correction for the transmission problem when the scatterer is a unit square and show it has an L^2-limit as $\epsilon \to 0$, provided that the boundary cutoff of cells is fixed. We also establish the $O(\epsilon^2)$ bulk correction describing the mean wave motion inside the scatterer. The analysis also highlights a previously established, yet scarcely recognized, fact that the $O(\epsilon)$ bulk correction of the mean motion vanishes identically.

24. P. Monk and V. Selgas, An inverse acoustic waveguide problem in the time domain, *Inverse Problems*, **32**, paper 055001 (2016).

    Abstract: We consider the problem of locating an obstacle in a waveguide from time domain measurements of causal waves. More precisely, we assume that we are given the scattered field due to point sources placed on a surface located inside the waveguide away from the obstacle, where the scattered field is measured on the same surface. From this multi-static scattering data we wish to determine the position and shape of an obstacle in the waveguide. To deal with this inverse problem, we adapt and analyze the time domain linear sampling method. This involves proving new time domain estimates for the forward problem, as well as analyzing several time domain

operators arising in the inversion scheme. We also implement the inversion algorithm and provide numerical results in two-dimensions using synthetic data.

25. D. Colton and Y.-J. Leung, The existence of complex transmission eigenvalues for spherically stratified media, *Applicable Analysis*, **96**, 39-47 (2016).

    Abstract: We continue our investigation of transmission eigenvalues for spherically stratified media. In particular, in the case when the index of refraction $n(r)$ is such that $n(1) \neq 1$, we establish conditions on $n(r)$ to guarantee that there exists an infinite number of complex transmission eigenvalues and when $n(1) = 1$, we establish very weak conditions on $n(r)$ to guarantee the existence of complex transmission eigenvalues.

26. D. Colton and Y.-J. Leung, On a transmission eigenvalue problem for a spherically stratified coated dielectric, *Inverse Problems and Imaging*, **10**, 369-377 (2016).

    Abstract: Suppose that the boundary of the unit ball in $\mathbb{R}^3$ is coated with a very thin layer of a highly conductive material and the refractive index $n(x)$ inside the ball is spherically stratified. We show that in this case the set of transmission eigenvalues behave quite differently than in the previous studied case of an uncoated ball. In particular, if the index of refraction varies smoothly across the boundary of the unit ball we show that complex eigenvalues always exist and accumulate on the real axis and that the real and complex eigenvalues uniquely determine the index of refraction without any restriction on its magnitude.

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1. F. Cakoni, D. Colton and S. Meng, The inverse scattering problem for a penetrable cavity with internal measurements, AMS Contemporary Mathematics, 615, 71-88 (2014).
2. F. Cakoni, Y. Hu and R. Kress, Simultaneous reconstruction of shape and general- ized impedance functions in electrostatic imaging, Inverse Problems, 30, paper 105010 (2014)
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Archival Publications:

9. F. Cakoni, S. Moskow and S. Rome, Perturbations of transmission eigenvalues for inho- mogeneous media in the presence of small inclusions, Inverse Problems and Imaging, 9, 725 - 748, (2015).
10. F. Cakoni, H. Haddar and H. Harris, Homogenization of the transmission eigenvalue problem for periodic media and application to the inverse problem, Inverse Problems and Imaging, 9, 1025 - 1049, (2015).
11. F. Cakoni, H. Haddar and S. Meng, Boundary integral equations for the transmission eigenvalue problem for Maxwell's equations, J. Int. Eqns. Appl., 27, 377-406, (2015).
12. D. Colton, Y.-J. Leung and S. Meng, Distribution of complex transmission eigenvalues for spherically stratified media, Inverse Problems, 31, paper 035006 (2015).
13. D. Colton. Y.-J. Leung and S.X. Meng, The inverse spectral problem for exterior trans- mission eigenvalues, Inverse Problems, 30, paper 055010 (2014).
14. D. Colton and S.X. Meng, Spectral properties of the exterior transmission eigenvalue problem, Inverse Problems, 30, paper 105010 (2014).

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15. Y. Guo, P. Monk and D. Colton, The linear sampling method for sparse small aperture data, Applicable Analysis, 95, 1599-1615 (2015).
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